Consider the polar equation $r = 3 + \cos 2\theta$. **SCORE: _____ / 27 PTS**

The following symmetry tests all fail: $(-r, \theta)$, $(-r, -\theta)$ **and** $(-r, \pi - \theta)$

[a] Run the other standard tests for symmetry for the polar equation, and summarize all conclusions in the table below.

$(r, -\theta)$: $r = 3 + \cos 2(-\theta)$

$r = 3 + \cos(-2\theta)$

$r = 3 + \cos 2\theta$

SYMMETRIC OVER POLAR AXIS

Type of symmetry	Conclusion
Over the pole	SYMMETRIC
Over the polar axis	SYMMETRIC
Over $\theta = \frac{\pi}{2}$	SYMMETRIC

$(r, \pi - \theta)$: $r = 3 + \cos 2(\pi - \theta)$

$r = 3 + \cos(2\pi - 2\theta)$

$r = 3 + \overset{1}{\cancel{\cos 2\pi}} \cos 2\theta + \overset{0}{\cancel{\sin 2\pi}} \sin 2\theta$

$r = 3 + \cos 2\theta$

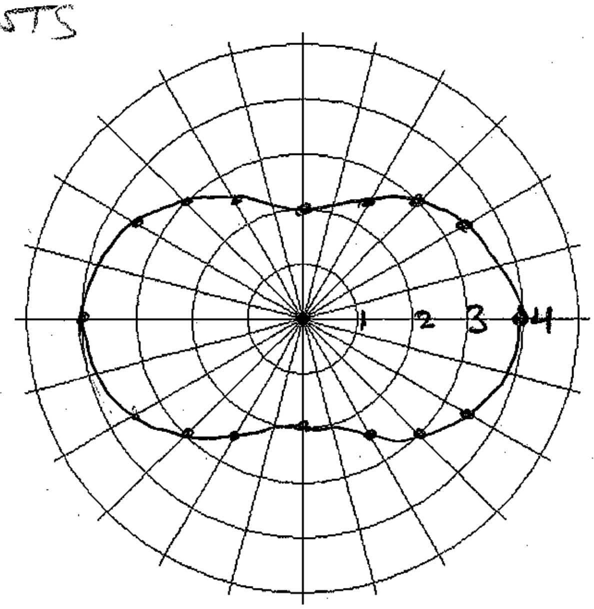
SYMMETRIC OVER $\theta = \frac{\pi}{2}$

AUTOMATICALLY SYMMETRIC OVER POLE

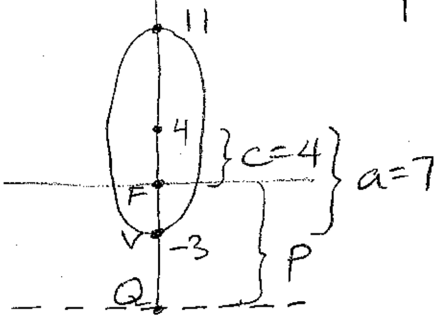
[b] Draw the graph of this polar equation by plotting points for as few θ – values as needed, and using symmetry to complete the graph. **List the polar co-ordinates (in ordered pair notation) of all points you needed to plot.**

ONLY NEED TO PLOT POINTS IN 1 QUADRANT

θ	r	
0	4	$(4, 0)$
$\frac{\pi}{6}$	3.5	$(3.5, \frac{\pi}{6})$
$\frac{\pi}{4}$	3	$(3, \frac{\pi}{4})$
$\frac{\pi}{3}$	2.5	$(2.5, \frac{\pi}{3})$
$\frac{\pi}{2}$	2	$(2, \frac{\pi}{2})$



Find the **polar** equation of the ellipse with vertices with **polar** co-ordinates $(11, \frac{\pi}{2})$ and $(3, \frac{3\pi}{2})$, and a focus at the pole. SCORE: _____ / 20 PTS



$r = \frac{ep}{1 - e\sin\theta}$

$11 = \frac{ep}{1 - e\sin\frac{\pi}{2}} \rightarrow 11 = \frac{ep}{1-e}$

$3 = \frac{ep}{1 - e\sin\frac{3\pi}{2}} \rightarrow 3 = \frac{ep}{1+e}$

$ep = 11 - 11e = 3 + 3e$

$8 = 14e$

$e = \frac{4}{7}$

$\frac{4}{7}p = 3 + 3(\frac{4}{7}) = \frac{33}{7}$

$p = \frac{33}{4}$

OR

CENTER $= (0, \frac{11-3}{2})$

$= (0, 4)$

$e = \frac{c}{a} = \frac{4}{7}$

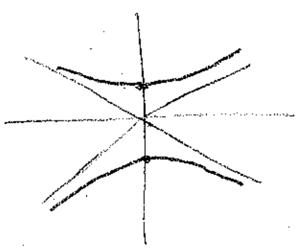
$e = \frac{VF}{VQ} = \frac{3}{p-3} = \frac{4}{7}$

$21 = 4p - 12$

$p = \frac{33}{4}$

$r = \frac{\frac{4}{7} \cdot \frac{33}{4}}{1 - \frac{4}{7}\sin\theta} = \frac{33}{7 - 4\sin\theta}$

Find the **rectangular** equation of the hyperbola with foci $(0, \pm 2)$ and asymptotes $y = \pm\frac{3}{4}x$. SCORE: _____ / 20 PTS



$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\frac{a}{b} = \frac{3}{4} \rightarrow a = \frac{3}{4}b$

$a^2 + b^2 = 2^2$

$(\frac{3}{4}b)^2 + b^2 = 4$

$\frac{25}{16}b^2 = 4$

$b^2 = \frac{64}{25}$

$b = \frac{8}{5}$

$a = \frac{6}{5}$

$\frac{y^2}{\frac{36}{25}} - \frac{x^2}{\frac{64}{25}} = 1$

OR

$\frac{25y^2}{36} - \frac{25x^2}{64} = 1$

Name the shapes of the following graphs. **SCORE: _____ / 21 PTS**

[a] the graph with equation $2+2x+3x^2+18y=0$

PARABOLA

[b] the graph with polar equation $r=-7-7\cos\theta$

CARDIOID $\left|\frac{-7}{-7}\right|=1$

[c] the graph with polar equation $r=\dfrac{15}{7-5\cos\theta}$

ELLIPSE $e=\frac{5}{7}$

[d] the graph with polar equation $\theta=2$

LINE

[e] the graph with equation $8+4x+2x^2+18y-3y^2=0$

HYPERBOLA

[f] the graph with polar equation $r=-5+2\cos\theta$

CONVEX LIMACON

5 $\left|\frac{-5}{2}\right| \geq 2$

[g] the locus of points in the plane that are six times as far from $(4,1)$ as they are from $x=7$

P F D

HYPERBOLA $PF = 6 * PQ \longrightarrow \frac{PF}{PQ} = 6 = e$

Consider the conic with rectangular equation $9x^2+4y^2+72x-8y+4=0$. **SCORE: _____ / 20 PTS**

[a] Find the co-ordinates of the vertex/vertices.

$$9x^2+72x+4y^2-8y=-4$$
$$9(x^2+8x)+4(y^2-2y)=-4$$
$$9(x^2+8x+16)+4(y^2-2y+1)=-4+144+4$$
$$9(x+4)^2+4(y-1)^2=144$$
$$\frac{(x+4)^2}{16}+\frac{(y-1)^2}{36}=1$$

ELLIPSE 

CENTER $(-4,1)$

VERTICES $(-4, 1\pm 6)$

$= (-4,7), (-4,-5)$

[b] Find the co-ordinates of the focus/foci.

$$c^2 = 36-16 = 20$$
$$c = 2\sqrt{5}$$

FOCI $(-4, 1\pm 2\sqrt{5})$

Convert the polar equation $r = 3 + \cos 2\theta$ to rectangular. **SCORE: _____ / 15 PTS**

$$r = 3 + \cos^2\theta - \sin^2\theta$$
$$r = 3 + \frac{x^2}{r^2} - \frac{y^2}{r^2}$$
$$r^3 = 3r^2 + x^2 - y^2$$
$$(x^2+y^2)^{\frac{3}{2}} = 3x^2 + 3y^2 + x^2 - y^2$$
$$= 4x^2 + 2y^2$$
$$(x^2+y^2)^3 = (4x^2+2y^2)^2$$

Find the zeros of the polar equation $r = 1 + 2\sin 2\theta$ for $\theta \in [0, 2\pi)$. **SCORE: _____ / 15 PTS**

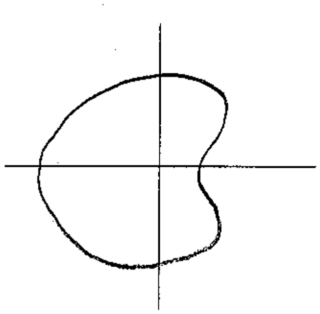
NOTE: You must solve this problem algebraically, NOT BY TRIAL & ERROR.

$$0 = 1 + 2\sin 2\theta$$
$$\sin 2\theta = -\frac{1}{2}$$
$$2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$$
$$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

$$0 \le \theta < 2\pi$$
$$0 \le 2\theta < 4\pi$$

Sketch the general shape, position and direction of the polar curve $r = 7 - 4\cos\theta$ on the axes on the right. **SCORE: _____ / 12 PTS**

NOTE: You do NOT need to find specific points. However, if the curve passes through the pole, it must be shown on the graph.



$1 < \left|\frac{7}{-4}\right| < 2 \longrightarrow$ LIMAÇON WITH DIMPLE

SYMMETRIC OVER X-AXIS

LARGER SIDE ON THE LEFT